NOTATION

I_λ = intensity of light of wavelength λ, energy/time-area
r = radial position in a tubular reactor, length
R = reactor radius, length
S_L = lamp source strength, energy/time-length
$\mu_{\lambda,c}$ = absorption coefficient of solution of concentration c for light of wavelength λ, length^{-1}

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Two-Phase Friction Factor for Nitrogen Between One Atmosphere and the Critical Pressure

JOHN D. ROGERS and GARY TIETJEN

Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

Flowing two-phase fluid systems exhibit enhanced pressure drop arising from the interaction of the phases. Martinelli, et al. (*4*) developed a model for predicting this pressure drop for no relative velocity between the phases. Wicks, et al. (*12*) compared this model to other models and to one developed by them and found it to rank high in its ability to correlate a broad spectrum of data. Bartlit and Williamson (*1*) made extensive analyses with the data bank of Wicks, et al. (*12*) and concluded that it was not possible to detect a flow pattern effect contributing to the scatter in two-phase pressure drop data except for annular flow which was found to lie slightly above all the other data. This effect was observed to be small. For these reasons, and since the Martinelli model is not explicitly dependent upon the flow pattern, this model was used to predict the two-phase pressure drop behavior of nitrogen. As a first approximation, because of the close similarity in properties, the correlation presented here can also be applied to two-phase air and oxygen transfer.

The methods of this paper follow those for parahydrogen developed by Rogers (*6*) and expanded upon in a subsequent presentation (*7*). The latter reference discusses the significance of the hydraulic parameters, α and β, of Martinelli and addresses itself to the nonlimitation of the model to an annular flow pattern as originally conceived.

The Martinelli, et al. (*3*, *4*) model yields the following rigorous equations for Φ, α, and β for turbulent-turbulent flow based upon the liquid phase.

$$\Phi = [\alpha^{0.25} + \beta^{0.25}/\chi^{0.75}]^{1.2} \tag{1}$$

$$\alpha = R_l^4 \; \Phi^{3.33} \tag{2}$$

$$\beta = R_g^4 \; (\Phi X)^{3.33} \tag{3}$$

where

$$\alpha = \frac{A_l}{\frac{\pi}{4} D_l^2} = \frac{R_l D_p^2}{D_l^2} \tag{4}$$

$$\beta = \frac{A_g}{\frac{\pi}{4} D_g^2} = \frac{R_g D_p^2}{D_g^2} \tag{5}$$

$$\chi = X^{1.11} = \left(\frac{\rho_g}{\rho_l}\right)^{0.555} \left(\frac{\mu_l}{\mu_g}\right)^{0.111} \left(\frac{1-x}{x}\right) \tag{6}$$

and

$$\Phi^2 = \left(\frac{\Delta P}{\Delta L}\right)_{TP} \Bigg/ \left(\frac{\Delta P}{\Delta L}\right)_l \tag{7}$$

Equation (1) can be rearranged to give

$$\beta = [\Phi^{0.833} - \alpha^{0.25}]^4 \, \chi^3 \tag{8}$$

From the correlation [see Figure 1, (*3*)], the values of α, β, and Φ at 1 atm. can be obtained. Martinelli and Nelson (*5*) have shown that the limiting case of Φ at the critical point is

$$\Phi = \left(\frac{1}{1-x}\right)^{0.9} = \left(\frac{\chi + 1}{\chi}\right)^{0.9} \tag{9}$$

This equation also applies every place in the liquid single phase region. At the critical point, $\rho_g = \rho_l$ and $\mu_g = \mu_l$ and can be substituted into the partial derivative with pressure at constant quality of Equation (9) and compared with the similar derivative of Equation (1). This comparison leads to

$$\left(\frac{\partial \alpha}{\partial P}\right)_x = 0 \tag{10}$$

and

$$\left(\frac{\partial \beta}{\partial P}\right)_x = 0 \tag{11}$$

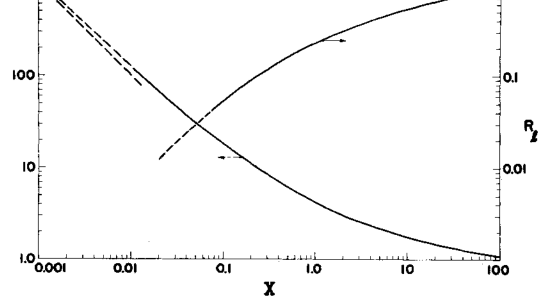

Fig. 1. Martinelli Φ and R_l at 1 atm. vs. X correlation.

along the single phase boundary and at the critical point condition. It can also be shown that

$$\alpha = 1 - x \tag{12}$$

and

$$\beta = x \tag{13}$$

$$\left(\frac{\partial \Phi}{\partial P}\right)_x = 0 \tag{14}$$

at the critical point condition.

These equations are sufficient to provide the end point conditions at constant quality for the values and slopes of Φ, α, and β, each versus the pressure, at both 1 atm. and the critical pressure. An estimate of α vs pressure shown in Figure 2 is then made over the entire pressure range from the knowledge of the end point conditions. Graphical values of $(\partial\alpha/\partial P)_x$ are taken from this figure. A plot of Φ vs. P is then estimated for each x = constant, and β vs. P is computed from Equation (8). $(\partial\beta/\partial P)_x$ is measured from the β vs. P graph. This with $(\partial\alpha/\partial P)_x$ is used to calculate $(\partial\Phi/\partial P)_x$ from the derivative of Equation (1). The result is compared with the graphical measurement of $(\partial\Phi/\partial P)_x$ from the first plot of Φ vs. P. The process is repeated until satisfactory convergence occurs. The reader is referred to (7) for details. The resulting Φ-X-P-x and R_l-X-P-x surfaces are shown in Figure 3.

To use this figure for an approximation for air or oxygen the parametric quality lines should be ignored and the pressure proportionately scaled to the existing pressure parameter lines between the 1 atm. curve and the critical pressure of these fluids.

Density data for the computations were taken from the work of Strobridge (9) and the viscosity data are from Rudenko (10) and co-workers (11), and Förster (2). The liquid-vapor coexistence curve of viscosity versus density of Rogers and Brickwedde (8) was interpolated from the saturated liquid region to the low pressure saturated gas region to obtain saturated vapor viscosities. The pressure derivatives of these properties were computed by a straight line difference technique.

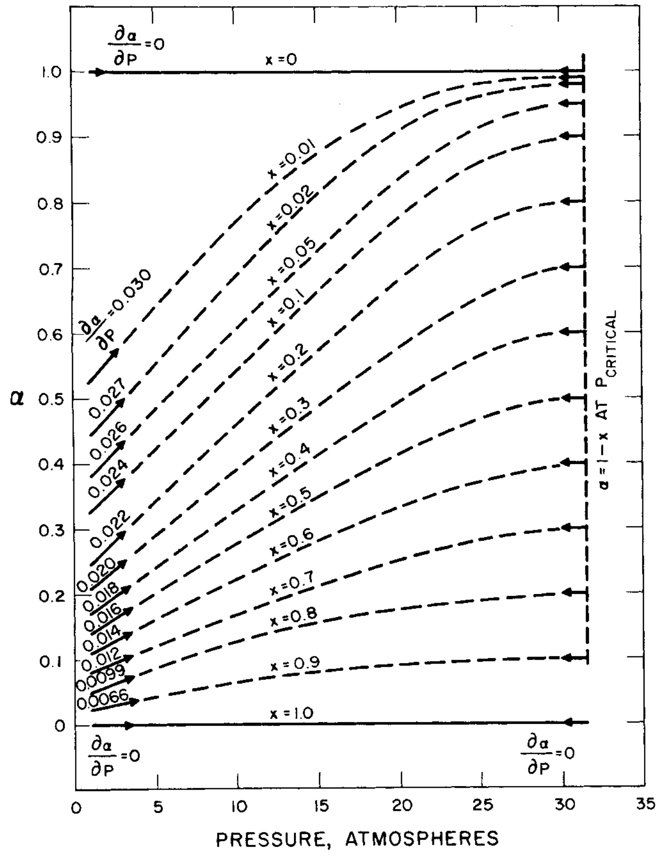

Fig. 2. α vs. pressure with quality as parameter.

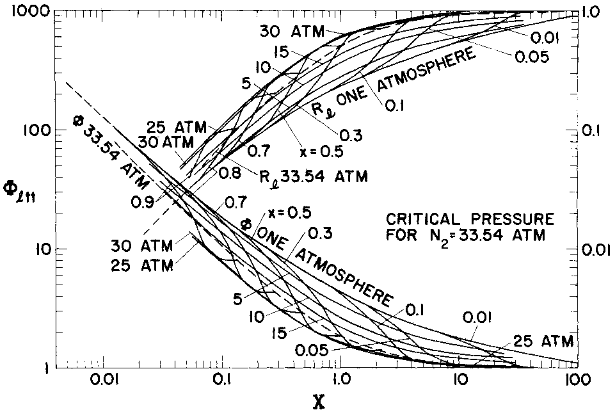

Fig. 3. Φ-X-P-x and R_l-X-P-x surfaces.

RESULTS

Two-phase friction factors and the volume fraction of the phases between one atmosphere and the critical point pressure have been obtained for nitrogen. The Martinelli, two-phase flow model was used in a manner which proved successful for parahydrogen (7). Values of the Martinelli Φ at pressures of 1, 2, 3, 5, 7, 10, 15, 25, and 30 atm. and qualities of 0.01, 0.02, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9 and one value at 20 atm. for x = 0.5 were computed. These 109 points were approximated by the following empirical equation,

$$\Phi^2 = \left(\frac{1}{1-x}\right)^{1.8} \{1 + x^{0.9245}[0.1072(33.54 - P) + 0.0038(33.54 - P)^3]/P^E\} \tag{15}$$

where $E = 9.963x - 52.77x^2 + 119.4x^3 - 122.3x^4 + 46.88x^5$. Of the 109 data points used, this equation gives 51 values of Φ^2 within 5%, 83 within 10%, 97 within 15%, 106 within 20%, and 109 within 21.5%. The equation has the correct single phase and critical point limiting values for Φ^2.

ACKNOWLEDGMENT

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NOTATION

A = cross-sectional area
D = hydraulic diameter of either phase or inside diameter of pipe; average effective dimension
g = geometry factor
L = length of pipe
P = pressure, atm.
R = volume fraction of either phase
x = weight fraction gas, quality
Φ = Martinelli two phase friction factor
X = $[(\rho_g/\rho_l)^{0.555}(\mu_l/\mu_g)^{0.111}(1 - x/x)]^{0.9}$ = Martinelli correlating parameter
α = liquid phase hydraulic parameter
β = gas phase hydraulic parameter
μ = viscosity of either phase
ρ = density of either phase
χ = $(\rho_g/\rho_l)^{0.555}(\mu_l/\mu_g)^{0.111}(1 - x/x)$ = Martinelli cor-

relating parameter

Subscripts

TP = two phase
g = gas or vapor phase
l = liquid phase
p = pipe
ltt = liquid phase flowing alone, both phases turbulent

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Axial Dispersion of Gases in Packed Beds

LESZEK Z. BALLA
Liquid Carbonic Corporation, Chicago, Illinois

and

THOMAS W. WEBER
State University of New York at Buffalo, Buffalo, New York

This study was prompted by some related work conducted here on the adsorption of gases in packed beds (*12, 16, 17*). In nearly all theoretical work on adsorption in these systems, it has been assumed that axial dispersion is small compared to bulk flow so that this dispersion could be neglected. A rather complex adsorption model was solved here using digital techniques and the results were compared with those obtained experimentally. The comparison was reasonably good, but some discrepancies were present. We felt it was desirable to measure the possible effect of axial dispersion in beds, either the same or similar to those used in the adsorption studies.

Axial dispersion in liquid systems has received considerable attention. A very comprehensive summary of this was presented by Chung and Wen (*9*). In contrast to liquid studies, relatively few have been made on gases. These are summarized in Table 1.

THEORY

Basically, two models have been used to describe the dispersion of a fluid in a packed bed, namely, the turbulent diffusive model and the mixing model. The mixing model pictures the voids of the bed as a series of well-agitated mixers as the fluid passes through. On the other hand, a perhaps more realistic picture is that the fluid particles are subjected to numerous changes in velocity, and to delays and trapping. The net result is a phenomena which may be described by a diffusional process superimposed upon a convective flow. These models have been discussed by Aris and Amundson (*2*), McHenry and Wilhelm (*15*), Levenspiel and Smith (*14*), Carberry (*6*), Carberry and Bretton (*7*), Bischoff and Levenspiel (*4*), and Sinclair and Potter (*20*). In particular, Aris and Amundson showed that for a point some distance from the bed entrance, the dispersion of a pulse by the two models would be identical.

In the present study, the diffusion model was chosen and the mathematical development and theory presented by Levenspiel (*13*), Levenspiel and Smith (*14*), and Carberry and Bretton (*7*) was followed. These treatments consider the case where a pulse of tracer is injected into a flowing stream at the entrance of the bed. The axial dispersion coefficient can then be calculated from the measured concentration at the exit. Two methods presented in these references were used for the calculation. The first involves the measured concentration corresponding to the average holdup time of the bed. The result is:

$$\left[\frac{CV_i}{Q}\right]_{\frac{vt}{V_i}=1} = \frac{1}{2\sqrt{\pi}}\left[\frac{D_L}{u_iL}\right]^{-1/2} \tag{1}$$

TABLE 1. SUMMARY OF STUDIES OF AXIAL DISPERSION OF GASES

Investigators	Gases used	Bed data	Experimental method
Deisler and Wilhelm (*10*)	H_2-N_2	column I.D. 1.1 in. ⅛ in. porous pellets	frequency response
McHenry and Wilhelm (*15*)	H_2-N_2 C_2H_4-N_2	column I.D. 1.94 in. 3 mm. glass spheres	frequency response
Carberry and Bretton (*7*)	Air-He	gas chromatographic column. 40-60 and 60-80 mesh particles.	pulse
Sinclair and Potter (*20*)	Hg. vapor-air	column I.D. 2 in. 0.0551 and 0.0172 in. glass spheres	frequency response
Chao and Hoelscher (*8*)	He-N_2	column I.D. 1 in. 0.1 in. (average) glass chips	pulse